

Sandwich Sudoku

This puzzle is, like Killer Sudoku, more than a variant of sudoku, and for the same reason. The digits in the answer, which is a sudoku array, all have a numerical significance, except the 1 and the 9. You are given not much more than the sum of the digits between the 1 and the 9 in every row and every column. Thus you have eighteen 'indirect givens' for every puzzle, and you need at least one 'direct' given in the body of the grid, otherwise you would not know the difference between 1 and 9 entries. A few more direct givens are useful for solving, and the general rule seems to be that the more such givens you have, the easier the puzzle will be. But beware, these puzzles are not easy!

Although this puzzle has been in the World Puzzle Championship for many years, it only burst into the consciousness of the general public in 2019, and in an article that year by Alex Bellos,

<https://www.theguardian.com/science/2019/may/06/can-you-solve-it-sandwich-sudoku-a-new-puzzle-goes-viral>

It has since got regular coverage under the name he invented for it; it had had the seductive name 'Between 1 and 9 Sudoku', but at least Alex was seduced. In his article he used the simpler puzzle, 'Between 1 and 6 Sudoku', to explain the ideas, and I pinch his example here.

The basic puzzle here is a sudoku with rectangular cages. The digits 1 to 6 must all appear once in each cage, once on each row, and once in each column. Sudoku rules thus apply.

Along the top row we see that the sum from the 1 to the point where the 6 must be is 10, and this is only possible with the available digits as $2 + 3 + 5 = 10$. These must be placed somewhere in the blue cells, with the remaining 4 at the end of the row.

1									10
									5
									5
									5
									5
									5
12	0	7	7	7	7	10			

1				6	4				10
									5
									5
									5
									5
									5
12	0	7	7	7	10				

The right-hand column similarly has 10 between 1 and 6, so we can add a 1 in the uppermost cage, with a 6 at the bottom. The left-hand column has a total of 12, so the digits between the markers must be $3 + 4 + 5 = 12$ leaving 2 at the bottom. The second set of blue cells therefore contains the same set as the first, with the green cells 3, 4 & 5.

1				6	4				10
								1	5
									5
									5
6									5
2								6	5
12	0	7	7	7	10				

The sum to 5 can be either $2 + 3$ or simply 5. In the second row it must be $2 + 3$ to set the 6 in the left-hand cage, whereas in the bottom row the 2 is already positioned outside the sandwich, so the entry must be a 5.

1				6	4				10
			6					1	5
									5
									5
6									5
2				1	5	6			5
12	0	7	7	7	10				

A 7 in the sandwich means either $2 + 5$ or $3 + 4$, so the position of the 1 in the penultimate column is determined.

The 4-th row then sets the 6 in the second column, and the 1 follows. We also know where the 5 must be in the top-right cage.

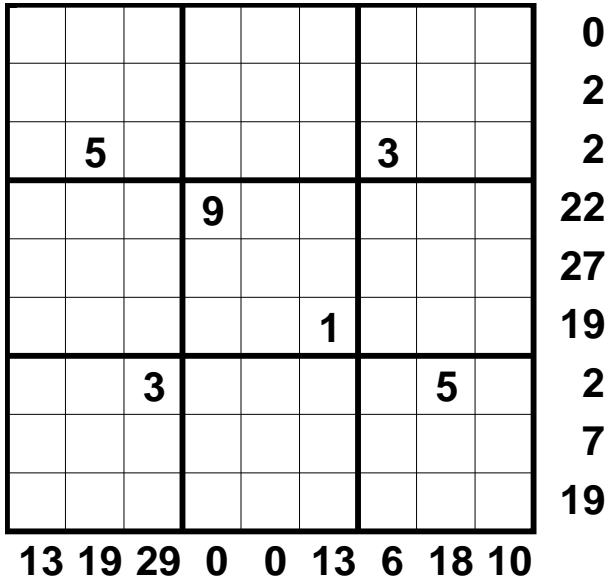
All the remaining 5s go into the yellow cells, and the rest is simply sudoku!

1			5	6	4				10
		6						1	5
									5
	1								5
	6			1					5
6				2					5
2				1	5	6			5
12	0	7	7	7	10				

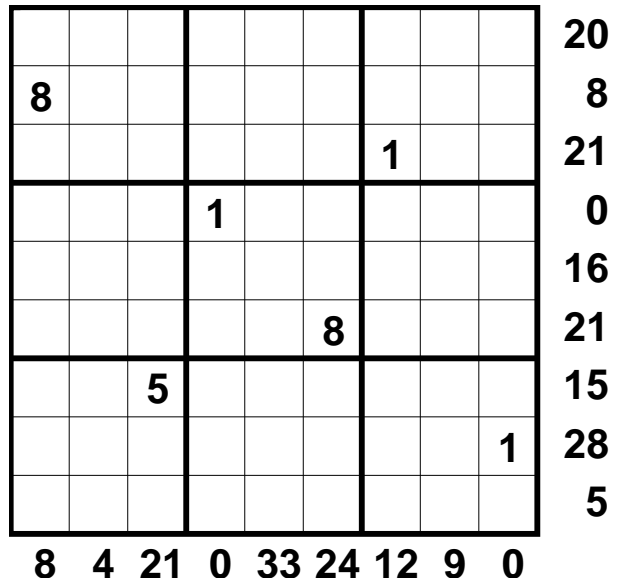
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		6						1	5
									5
	1		6						5
	6			1					5
6		1		2					5
2				1	5	6			5
12	0	7	7	7	10				

Now there is a steep learning curve for tackling the 9×9 . When you are lost for what to do, check the sandwiches! You can learn also by trying to make these puzzles, and then become frustrated by the multiple solutions that abound. Furthermore don't trust the uniqueness of the examples you may chance upon, though of course all the examples here are stonebonker unique!

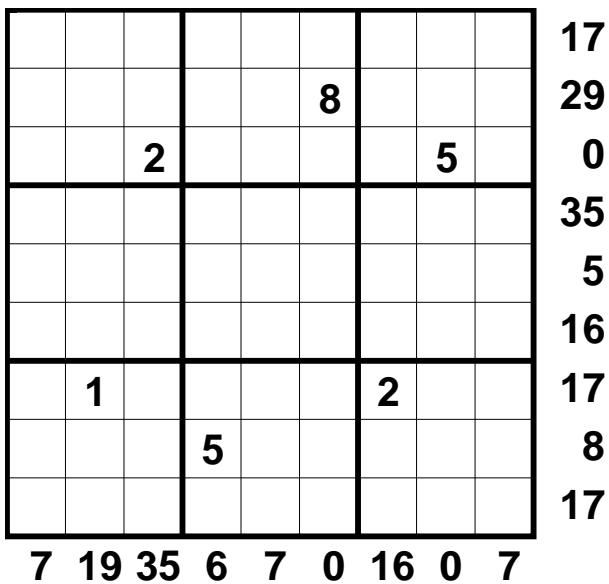
Sandwich Sudoku



1

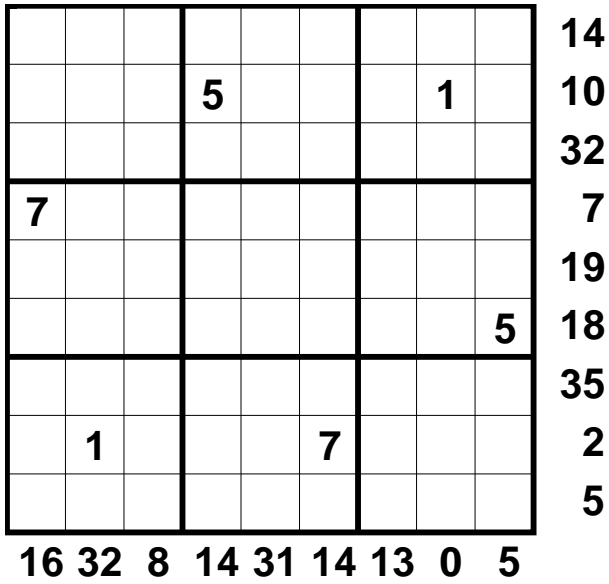


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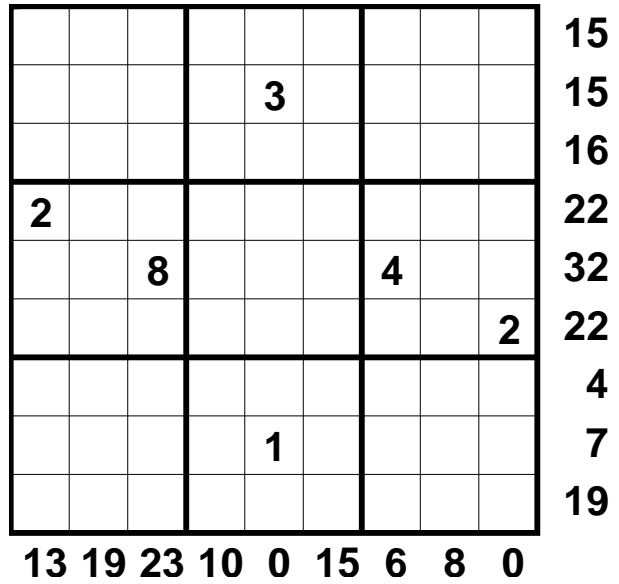


3

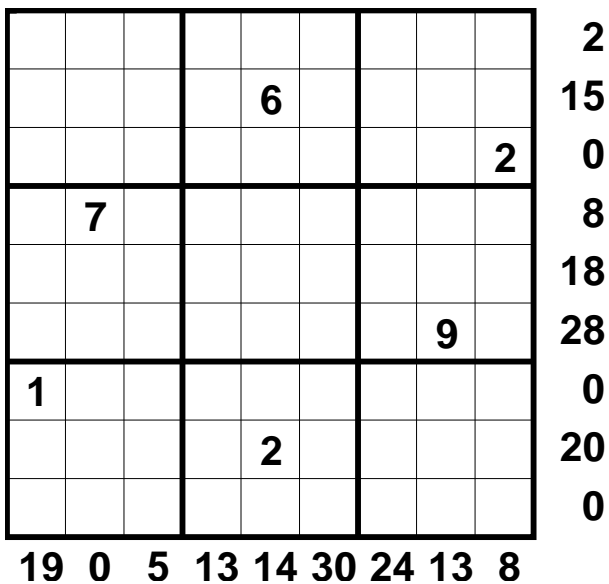
Sandwich Sudoku



4

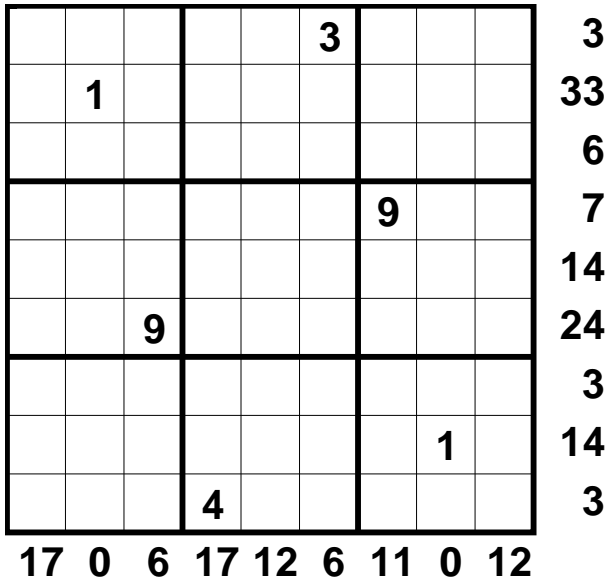


5

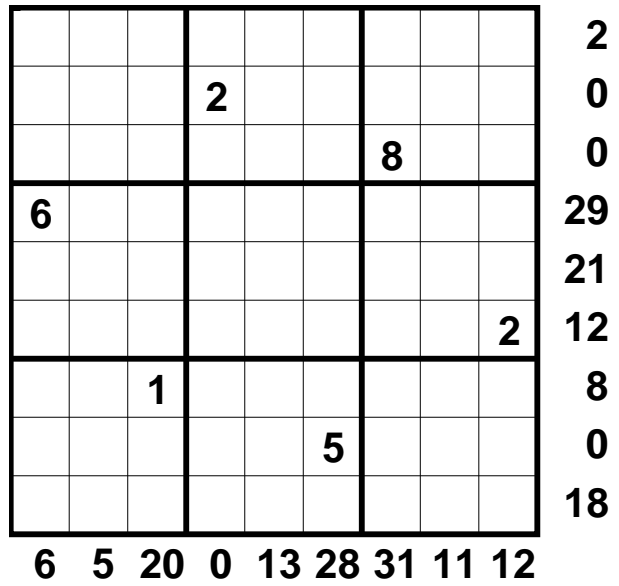


6

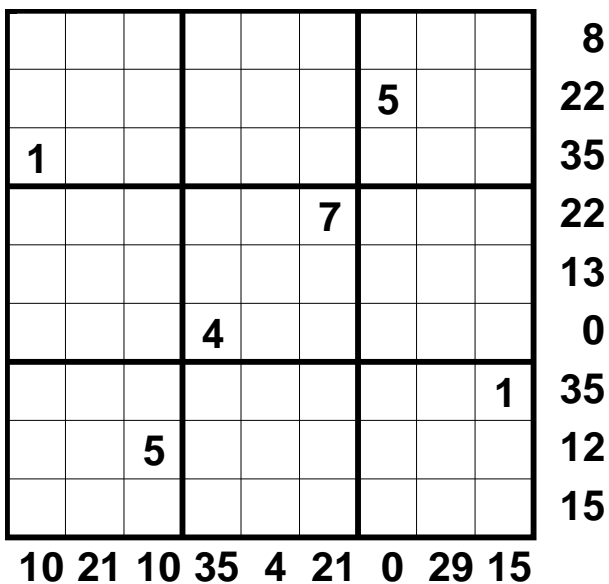
Sandwich Sudoku



7



8



9

Sandwich Sudoku

		3				
						1
		2				
1						
				4		

4
0
0
14
17
12
0
9
19

10

21 15 16 2 29 2 3 4 25

	2					5
	9					
		7	2			
						3
	5					2

35
4
4
14
0
0
0
9
19

11

22 15 18 2 28 0 8 4 35

						2
		7				
						9
9						
					1	
7						

2
9
13
0
5
10
6
0
7

12

6 0 18 0 9 15 31 21 12

Sandwich Sudoku

		1			4			
1								
							6	
			4			1		

2
0
0
30
11
11
6
0
7

13

10 5 27 0 2 21 31 13 12

						2		
				4				
9							7	
					1			
			2					

10
2
3
27
27
5
6
19
19

14

23 19 8 0 29 9 6 8 0

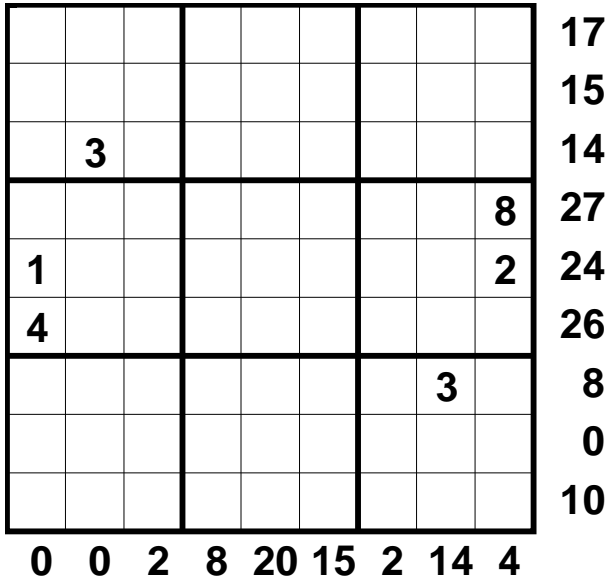
				1				
								2
3								
							9	
1								
				2				

0
0
0
23
15
21
6
20
20

15

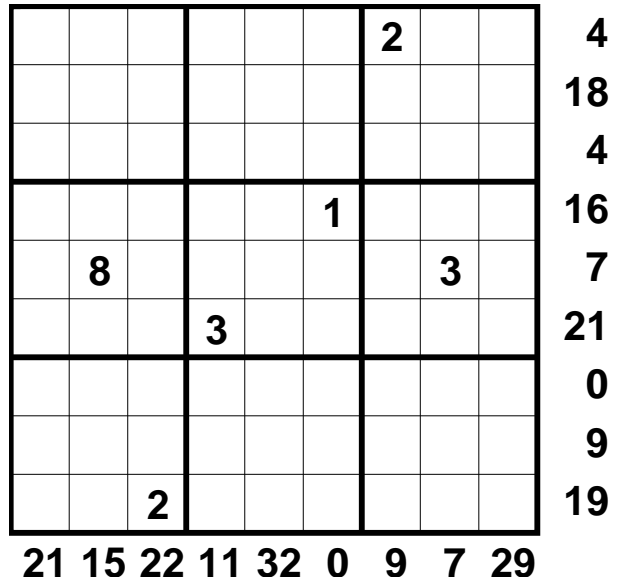
12 35 16 0 11 6 24 13 8

Sandwich Sudoku

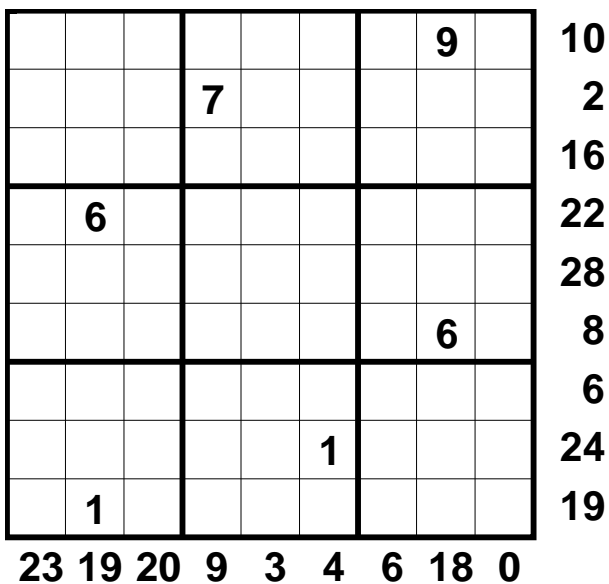


16

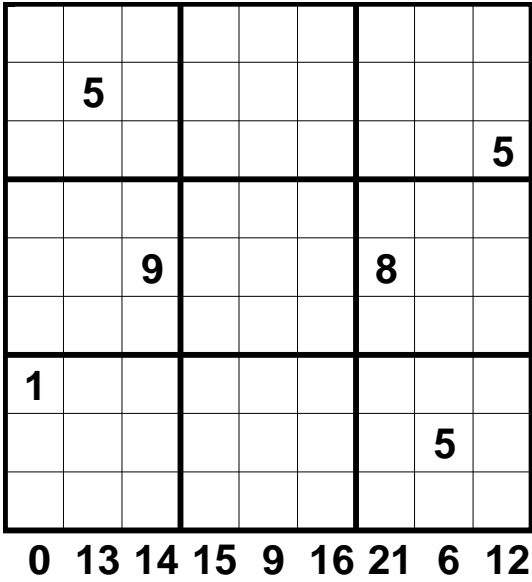
17



18

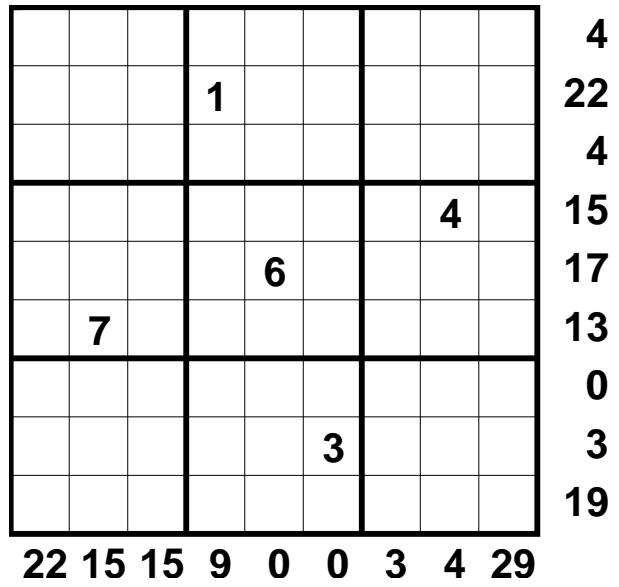


Sandwich Sudoku



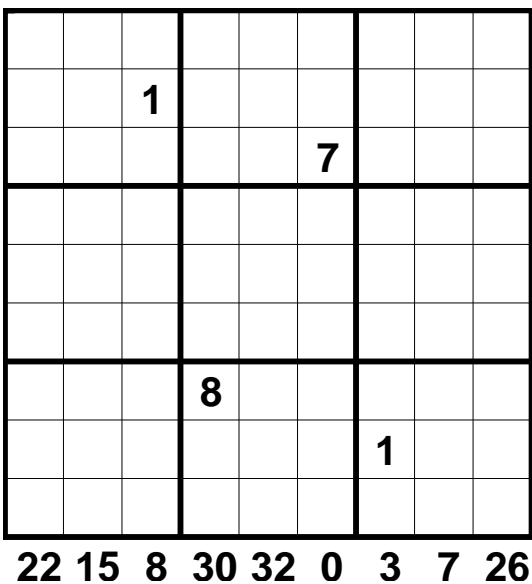
25
10
7
25
27
0
7
28
9

19



4
22
4
15
17
13
0
3
19

20



11
4
0
15
8
0
0
9
19

21

Sandwich Sudoku

		1						
					1			
		2						
						4		

4
0
4
26
35
3
6
18
21

22

13 28 3 12 0 7 24 20 19

4						8		
		4						1

0
28
11
20
35
6
2
2
7

23

5 28 15 12 3 17 28 6 7

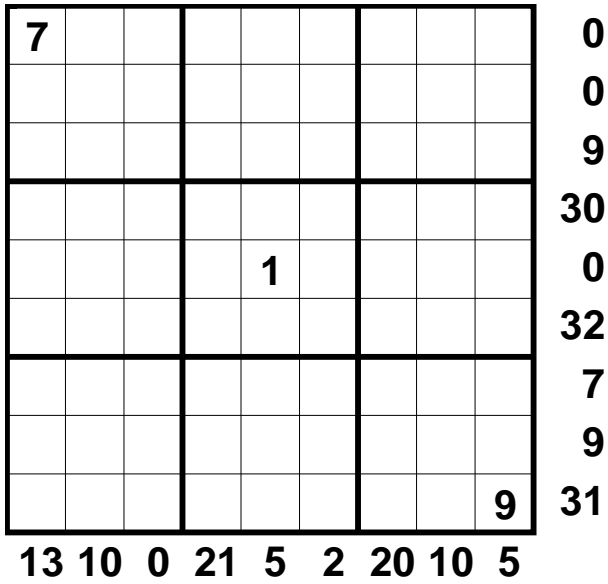
6								
					1			
								1

0
0
5
30
0
32
0
12
31

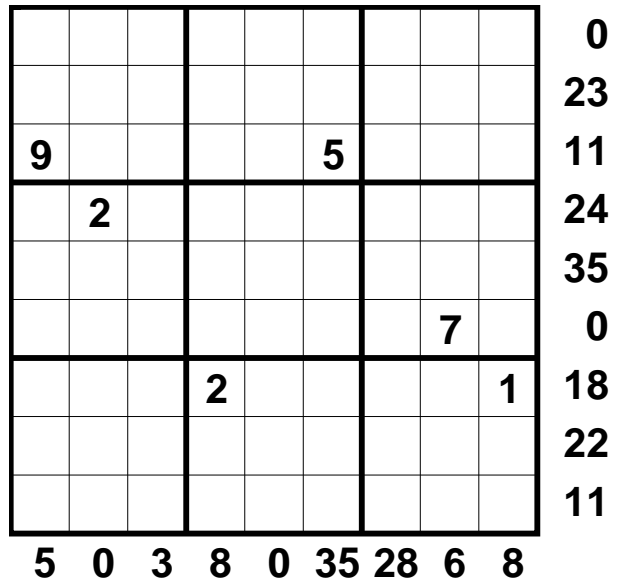
24

13 10 0 6 12 12 20 13 8

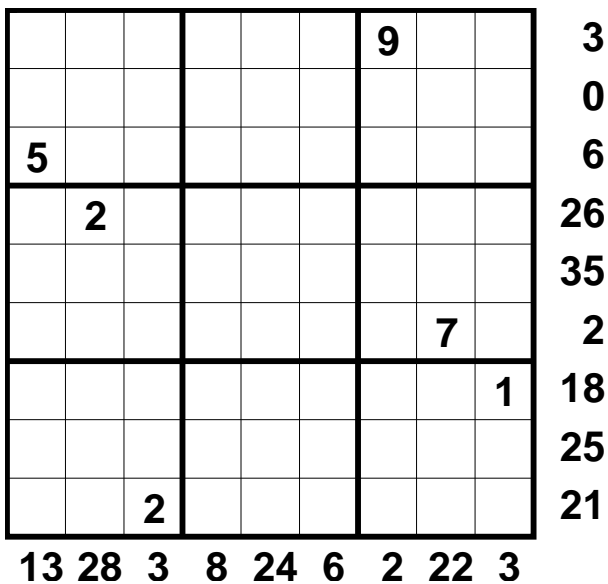
Sandwich Sudoku



25

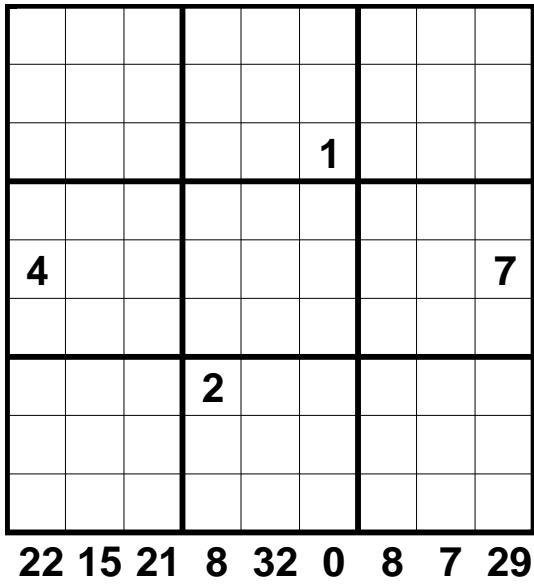


26



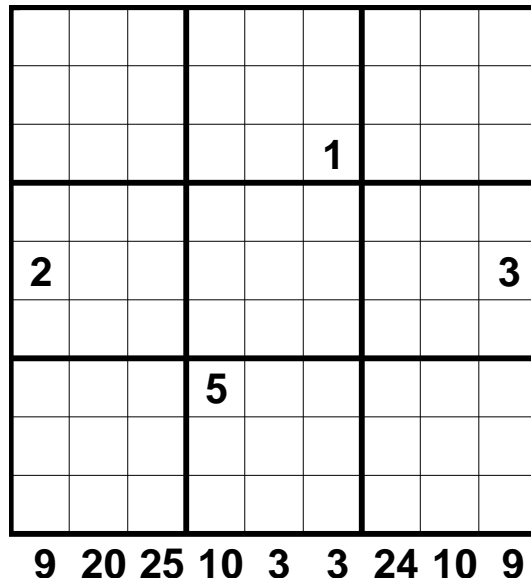
27

Sandwich Sudoku



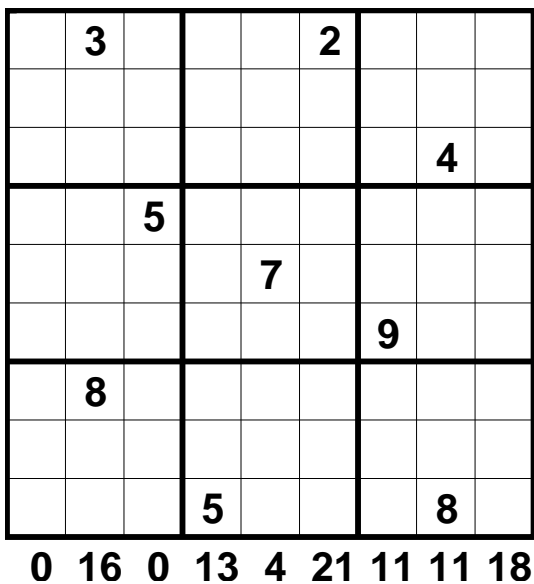
4
18
4
15
7
23
0
9
19

28



0
3
9
30
6
32
8
2
9

29



8
26
7
3
27
26
35
13
13

30

Sandwich Sudoku solutions 1 – 15

7	3	4	6	5	8	2	1	9
1	2	9	3	7	4	5	8	6
8	5	6	1	2	9	3	7	4
2	6	5	9	8	7	4	3	1
3	1	8	2	4	6	7	9	5
9	4	7	5	3	1	8	6	2
6	7	3	4	1	2	9	5	8
4	8	1	7	9	5	6	2	3
5	9	2	8	6	3	1	4	7

1	6	4	8	2	9	5	7	3
8	5	3	7	1	6	2	9	4
9	2	7	4	5	3	1	6	8
4	7	9	1	6	2	8	3	5
2	3	8	9	7	5	4	1	6
5	1	6	3	4	8	9	2	7
6	4	5	2	3	1	7	8	9
7	9	2	6	8	4	3	5	1
3	8	1	5	9	7	6	4	2

3	5	1	7	6	4	9	2	8
6	9	7	2	5	8	3	4	1
4	8	2	1	9	3	6	5	7
1	2	8	6	4	5	7	3	9
7	6	4	9	3	2	1	8	5
9	3	5	8	1	7	4	6	2
5	1	6	3	8	9	2	7	4
2	4	3	5	7	1	8	9	6
8	7	9	4	2	6	5	1	3

2	9	7	3	4	1	6	5	8
4	6	3	5	9	8	2	1	7
1	8	5	7	2	6	4	9	3
7	4	1	2	5	9	3	8	6
3	5	8	1	6	4	7	2	9
6	2	9	8	7	3	1	4	5
9	7	4	6	8	2	5	3	1
5	1	2	9	3	7	8	6	4
8	3	6	4	1	5	9	7	2

7	3	6	9	5	8	2	1	4
9	2	4	6	3	1	7	8	5
8	5	1	4	7	2	3	9	6
2	6	5	1	4	7	8	3	9
3	9	8	5	2	6	4	7	1
1	4	7	3	8	9	5	6	2
6	7	3	2	9	4	1	5	8
4	8	9	7	1	5	6	2	3
5	1	2	8	6	3	9	4	7

7	8	1	2	9	5	4	3	6
9	4	2	3	6	1	7	5	8
5	6	3	4	8	7	9	1	2
2	7	9	8	1	3	5	6	4
8	3	6	9	5	4	2	7	1
4	1	5	6	7	2	8	9	3
1	9	8	7	4	6	3	2	5
3	5	7	1	2	8	6	4	9
6	2	4	5	3	9	1	8	7

5	4	7	8	9	3	1	2	6
2	1	3	6	5	4	7	8	9
8	9	6	1	7	2	4	5	3
3	6	8	5	1	7	9	4	2
1	2	5	3	4	9	8	6	7
4	7	9	2	8	6	5	3	1
6	5	4	7	2	1	3	9	8
7	8	2	9	3	5	6	1	4
9	3	1	4	6	8	2	7	5

8	6	4	7	5	3	1	2	9
7	1	9	2	8	4	6	3	5
3	5	2	6	1	9	8	4	7
6	9	7	5	4	2	3	8	1
5	2	3	1	6	8	7	9	4
1	4	8	9	3	7	5	6	2
4	7	1	8	9	6	2	5	3
2	3	6	4	7	5	9	1	8
9	8	5	3	2	1	4	7	6

7	5	3	9	8	1	2	6	4
8	2	9	6	7	4	5	1	3
1	4	6	3	5	2	8	7	9
3	9	4	5	6	7	1	8	2
5	6	1	2	3	8	9	4	7
2	8	7	4	1	9	3	5	6
9	7	2	8	4	5	6	3	1
6	1	5	7	9	3	4	2	8
4	3	8	1	2	6	7	9	5

9	4	1	3	6	5	2	8	7
7	2	8	4	9	1	6	5	3
3	6	5	8	7	2	4	9	1
2	1	3	6	5	9	7	4	8
4	8	9	7	2	3	5	1	6
5	7	6	1	4	8	9	3	2
1	9	4	2	8	7	3	6	5
8	5	7	9	3	6	1	2	4
6	3	2	5	1	4	8	7	9

1	4	3	6	7	5	2	8	9
8	2	9	4	1	3	6	5	7
5	6	7	8	2	9	4	1	3
2	9	6	3	5	1	7	4	8
3	8	5	7	4	2	1	9	6
4	7	1	9	6	8	5	3	2
9	1	4	2	8	7	3	6	5
7	5	8	1	3	6	9	2	4
6	3	2	5	9	4	8	7	1

6	4	5	7	8	3	9	2	1
8	9	7	2	1	4	6	3	5
3	1	2	6	5	9	8	4	7
5	2	3	8	4	6	7	1	9
7	8	1	5	9	2	3	6	4
9	6	4	1	3	7	5	8	2
4	5	8	9	6	1	2	7	3
2	3	6	4	7	5	1	9	8
1	7	9	3	2	8	4	5	6

4	6	8	7	5	3	9	2	1
7	9	1	2	8	4	6	3	5
3	5	2	6	1	9	8	4	7
5	1	4	3	2	6	7	8	9
6	2	7	5	9	8	3	1	4
9	8	3	1	4	7	5	6	2
8	4	5	9	6	1	2	7	3
2	3	6	4	7	5	1	9	8
1	7	9	8	3	2	4	5	6

7	3	6	5	9	8	2	1	4
1	2	9	3	4	6	7	8	5
4	5	8	2	7	1	3	9	6
2	6	1	4	8	7	5	3	9
8	9	5	6	3	2	4	7	1
3	4	7	1	5	9	8	6	2
6	7	3	9	2	4	1	5	8
9	8	4	7	1	5	6	2	3
5	1	2	8	6	3	9	4	7

7	1	9	2	8	5	4	3	6
2	4	6	3	1	9	7	5	8
5	8	3	7	4	6	9	1	2
9	3	5	8	7	1	2	6	4
8	6	2	4	9	3	5	7	1
4	7	1	6	5	2	8	9	3
1	2	4	9	6	7	3	8	5
3	5	7	1	2	8	6	4	9
6	9	8	5	3	4	1	2	7

Sandwich Sudoku solutions 16 – 30

7	1	6	5	2	4	9	8	3
5	9	8	7	1	3	2	4	6
2	3	4	9	8	6	1	7	5
9	6	5	2	4	7	3	1	8
1	8	7	6	3	9	4	5	2
4	2	3	1	5	8	7	6	9
6	7	1	8	9	2	5	3	4
8	4	2	3	7	5	6	9	1
3	5	9	4	6	1	8	2	7

1	4	9	7	3	5	2	8	6
3	2	8	4	1	6	7	5	9
7	6	5	2	8	9	4	1	3
2	9	3	6	7	1	5	4	8
4	8	6	9	5	2	1	3	7
5	7	1	3	4	8	6	9	2
9	1	4	8	2	7	3	6	5
8	5	7	1	6	3	9	2	4
6	3	2	5	9	4	8	7	1

7	3	6	4	1	8	2	9	5
1	2	9	7	3	5	4	8	6
4	5	8	2	9	6	3	7	1
2	6	5	1	4	7	8	3	9
3	9	7	6	8	2	5	1	4
8	4	1	3	5	9	7	6	2
6	7	3	9	2	4	1	5	8
9	8	4	5	7	1	6	2	3
5	1	2	8	6	3	9	4	7

2	9	4	6	7	5	3	1	8
3	5	1	2	8	9	7	4	6
7	8	6	1	4	3	9	2	5
6	1	8	3	5	7	2	9	4
5	3	9	4	2	6	8	7	1
4	7	2	8	9	1	5	6	3
1	2	5	9	3	4	6	8	7
9	4	3	7	6	8	1	5	2
8	6	7	5	1	2	4	3	9

1	4	9	7	3	5	2	8	6
8	2	3	1	4	6	7	5	9
5	6	7	2	8	9	4	1	3
2	9	5	3	7	1	6	4	8
3	8	1	4	6	2	5	9	7
4	7	6	9	5	8	1	3	2
9	1	4	8	2	7	3	6	5
7	5	8	6	1	3	9	2	4
6	3	2	5	9	4	8	7	1

9	4	7	1	3	5	2	8	6
8	2	1	4	9	6	7	5	3
3	6	5	2	8	7	4	9	1
2	1	3	7	5	9	6	4	8
4	8	9	6	2	1	5	3	7
5	7	6	3	4	8	9	1	2
1	9	4	8	7	2	3	6	5
7	5	8	9	6	3	1	2	4
6	3	2	5	1	4	8	7	9

2	7	6	5	8	3	9	4	1
4	9	1	6	2	7	3	5	8
5	8	3	9	4	1	2	6	7
3	2	9	8	7	5	6	1	4
1	5	7	4	6	2	8	3	9
6	4	8	1	3	9	5	7	2
7	3	4	2	9	6	1	8	5
9	6	5	7	1	8	4	2	3
8	1	2	3	5	4	7	9	6

2	7	6	3	8	9	1	5	4
4	1	5	6	2	7	8	9	3
9	8	3	1	4	5	2	6	7
5	2	9	4	7	3	6	1	8
1	6	7	8	5	2	3	4	9
3	4	8	9	6	1	5	7	2
7	3	1	2	9	6	4	8	5
6	5	4	7	3	8	9	2	1
8	9	2	5	1	4	7	3	6

6	2	3	7	4	5	1	9	8
5	4	9	1	6	8	2	7	3
7	8	1	2	3	9	5	6	4
9	6	7	4	8	2	3	1	5
2	5	4	9	1	3	6	8	7
3	1	8	6	5	7	4	2	9
8	3	6	5	7	1	9	4	2
1	7	2	3	9	4	8	5	6
4	9	5	8	2	6	7	3	1

7	2	3	6	4	5	9	1	8
5	4	1	9	3	8	2	7	6
6	8	9	2	7	1	5	3	4
1	6	7	4	8	2	3	9	5
2	5	4	3	1	9	6	8	7
3	9	8	7	5	6	4	2	1
8	3	6	5	9	7	1	4	2
9	7	2	1	6	4	8	5	3
4	1	5	8	2	3	7	6	9

2	6	7	8	3	9	1	4	5
4	5	1	6	2	7	8	9	3
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