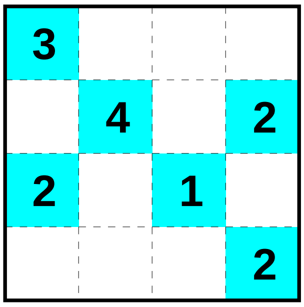
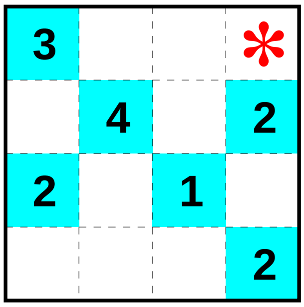
Searchlights

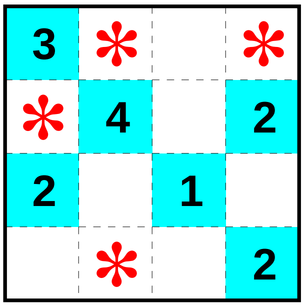
The aim is to place searchlights in some of the empty cells of a grid. Some cells are blocked off, here in light blue. Each block cell contains a number, a given, which tells you how many lights shine on that particular cell, shining horizontally or vertically. A light can shine through another light, but not through the block cells. A cell can have at most one light.



Consider the puzzle on the left.

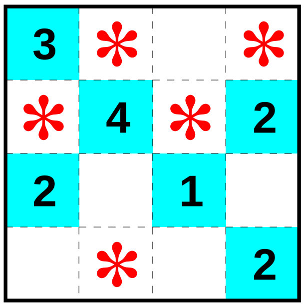
The [2] in the second row has three possible positions for a light, but as two of these shine onto the [1], only one of these is permissable, therefore the third possible position must contain a light (see right).





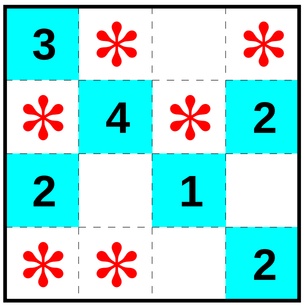
A similar argument can be applied to the [4], which has five possible positions for lights and again two of these shine on the [1], so we can immediately place three more lights which do not shine on the [1] (see left).

We can now find the light which shines on [1].



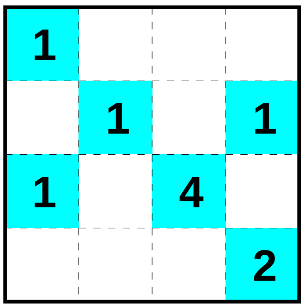
Of the three positions which we have seen that could shine on the [1], only one shines on both the [2] that we have investigated and the [4], so this must be the final light to satisfy the [2] and the [4].

The [3] is already satisfied, so there are just two more [2]s to satisfy, and this can only be done with a light in the bottom left corner. The solution is now complete.



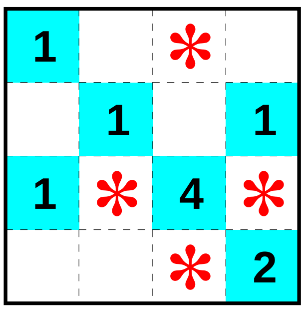
This puzzle features in Alex Bellos' *Puzzle Ninja* book, but I was rather frustrated when it took me ages to solve the last of his puzzles. The most remarkable thing about this last puzzle is that it has definitely a unique solution. When making Searchlights puzzles for this web-site I have found it very difficult to make unique solution puzzles, so I have made only a few for the keen puzzlers.

I should have realised there would be problems as all the examples of this puzzle have every block cell containing a given. Usually when one makes a new example of some puzzle type and there are multiple solutions, the way to rectify this is to add one or two more givens. Here rectification is simply not possible except either by shuffling the searchlights or adding more block cells.



This particular puzzle briefly poses a nice mathematical question. The puzzle on the left is the 'inverse' of the example puzzle above, with lights placed only in the cells which previously were empty. The solution, on the right, is a unique solution. Is there a theorem here?

All will be revealed!



Searchlights 7×7

	2				2	
3			3			4
		3		2		
	4		1		1	
		4		2		
3			2			3
	3				1	

1

		1		1		
	5				3	
3			4			2
		5		4		
3			2			1
	6				5	
		3		2		

2

		1				3
1			2		2	
		1		3		
2			2			2
		3		1		
	4		2			2
1				3		

3

	3			2		
		3				1
3			2		2	
		2		1		
	5		1			1
1				2		
		3			2	

4

		2		3		
	2				3	
1			1			3
		3		3		
3			4			2
	3				3	
		2		2		

5

		2		4		
2			3			4
	2				2	
		1		6		
	3				3	
4			3			4
		1		3		

6

		3				1
	1		2		1	
4			2			1
	2	1		3	1	
2			2			3
	2		2		3	
2				4		

7

		2			3	
2			3			
		2		2		2
	4		2		4	
5		2		1		
			2			2
	2			1		

8

Searchlights 8×8

2				4		1	
	1		3				2
		5			3		
	2		4				4
4				7		2	
		2			2		
2				1		2	
	3		3				3

	3				3		
		3					2
	1			1		3	
			1		1		4
2		1		2			
	1		2			2	
5					4		
		3				3	

1 **2**

1				3			3
			3				
		2			1		
3				4		5	
	2		1				3
		4			4		
				3			
2			2				3

1				1		2	
		3					1
2					2		
	3		2				
				4		2	
		1					1
2					4		
	1		4				3

3 **4**

	3			3		2	
1			2				
		2			3		
	2		4				2
1				2		3	
		4			2		
				3			0
	2		3			2	

		1			2		1
2				4			
	2		1			1	
		2		2			2
2			1		3		
	1			2		2	
			4				1
1		1			3		

5 **6**

Searchlights 9×9

1		4			4			
				2		1		1
0			3				1	
	3				4			
2		2		1		2		2
			3				2	
	3				3			1
3		1		2				
			2			2		1

1

4				2				1
	2		3		3		3	
		2		2		4		
	2						3	
3		1		2		2		1
	2						2	
		3		2		1		
	1		4		4		2	
1				2				1

2

1		1		2		2		1
			3		2			
3		2				1		1
	3			3			3	
			4		3			
	5			3			2	
3		4				3		3
			4		2			
1		2		1		1		2

3

	1			2			2	
2		1				3		1
	2			1			2	
			3		4			
3		3				4		1
			6		1			
	3			3			2	
2		3				3		1
	2			1			1	

4

2			2		2			4
	2			4			4	
			3		2			
	2						3	
2		6		2		4		4
	2						1	
			4		2			
	4			3			2	
2			1		2			2

5

		2		3			2	
3			6			2		
	2				4			3
		3					5	
2				2				2
	4					4		
1			4				3	
		2			4			1
	1			0		1		

6

Searchlights 7 × 7 Solutions

	2	*			2	
3	*		3	*	*	4
*		3		2		*
	4	*	1		1	
*	*	4		2		*
3	*		2	*		3
	3	*			1	

1

	*	1		1		
*	5		*		3	*
3	*		4	*		2
*	*	5	*	4	*	
3		*	2			1
*	6	*		*	5	*
	*	3		2	*	

2

		1		*		3
1	*		2		2	*
		1	*	3		*
2	*		2			2
*	*	3		1		
	4	*	2	*		2
1				3	*	*

3

	3	*		2	*	
*		3	*			1
3	*		2		2	
	*	2		1	*	
*	5	*	1			1
1				2		*
	*	3		*	2	

4

	*	2		3	*	*
	2	*			3	
1			1	*		3
*		3		3		*
3		*	4		*	2
*	3		*	*	3	
		2	*	2		

5

	*	2	*	4	*	*
2			3	*		4
*	2				2	
		1	*	6	*	*
*	3			*	3	*
4	*		3	*		4
*		1		3		*

6

*		3	*			1
*	1		2	*	1	
4		*	2	*		1
*	2	1		3	1	
2	*		2		*	3
	2		2	*	3	*
2	*		*	4		*

7

	*	2	*		3	
2			3		*	*
*		2		2	*	2
*	4	*	2	*	4	
5	*	2		1		
*			2		*	2
*	2			1		*

8

Searchlights 8 × 8 Solutions

1

2		*		4		1	
	1	*	3	*	*		2
*		5		*	3		
	2		4	*		*	4
4	*	*	*	7	*	2	
	*	2			2		*
2			*	1		2	*
*	3		3			*	3

2

*	3	*			3	*	
	*	3	*				2
	1			1	*	3	*
			1		1		4
2		1		2			*
*	1		2			2	*
5		*	*	*	4	*	
*		3	*			3	*

3

1		*		3		*	3
			3	*		*	*
		2			1		
3	*			4	*	5	*
	2	*	1				3
*		4		*	4		*
*		*		3			
2			2		*	*	3

4

1				1	*	2	*
*	*	3					1
2					2		
*	3		2				
	*	*	*	4	*	2	
		1					1
2			*		4		*
	1		4		*	*	3

5

*	3		*	3	*	2	
1			2				
	*	2	*		3		
	2		4	*		*	2
1				2	*	3	
*	*	4			2	*	
		*	*	3			0
	2	*	3	*		2	

6

	*	1			2		1
2	*			4	*	*	*
	2		1			1	
*		2	*	2	*		2
2			1		3		
	1			2	*	2	
*	*		4	*			1
1		1	*		3	*	

Searchlights 9 × 9 Solutions

1

1	*	4	*	*	4	*		
		*		2		1		1
0			3		*		1	
	3		*		4			*
2	*	2		1	*	2		2
*			3				2	*
	3		*		3			1
3	*	1		2	*	*		
*			2			2	*	1

2

4	*		*	2				1
*	2		3		3	*	3	*
*		2	*	2	*	4	*	
	2		*			*	3	
3		1		2	*	2		1
	2	*		*			2	
*		3		2		1		
	1	*	4	*	4	*	2	
1				2			*	1

3

1		1	*	2	*	2	*	1
*			3		2		*	
3	*	2				1		1
	3		*	3		*	3	
	*		4	*	3			*
*	5	*	*	3			2	
3	*	4			*	3	*	3
*		*	4		2			*
1		2	*	1		1	*	2

4

*	1			2	*	*	2	
2		1	*			3		1
*	2			1			2	*
			3	*	4	*	*	
3	*	3	*			4		1
*		*	6		1	*		
	3		*	3		*	2	
2	*	3	*			3		1
	2		*	1			1	*

5

2		*	2		2		*	4
	2	*	*	4	*	*	4	*
*			3		2			*
	2	*				*	3	*
2		6	*	2		4		4
	2					*	1	
	*	*	4	*	2			
*	4	*		3	*		2	*
2	*		1		2	*		2

6

		2	*	3	*		2	*
3	*	*	6		*	2		*
*	2				4			3
		3	*	*			5	*
2				2			*	2
*	4		*		*	4	*	
1			4		*		3	
	*	2	*		4	*		1
	1			0		1		

Investigating the possible mathematics of this particular puzzle one is tempted to try solving the inverses of the available puzzles, of which I could find five.

0		2		4	
					5
3			7		
		4			3
5					
	4		1		1

1			5			2
	6				6	
		4		3		
2			6			3
		3		3		
	6				5	
3			5			3

	5		2		3		
5						11	
			3				2
0		6		5			
			5		5		2
4				4			
	10						4
		4		1		5	

		5					6	
	6				6			4
6			8			6		
		6					8	
				15				
	8					6		
		6			9			7
3			7				5	
	4					5		

3				5					5
		6				5			
				8				6	
	4				8				
			9				5		4
4		8				11			
				9				8	
	6				9				
			5				7		
6					7				6

As these are not my puzzles I feel that I should not disclose the solutions. However, the important fact for mathematics is that the first four have unique solutions, but sadly there is no theorem as the last puzzle has 54 solutions. Well, I think there are 54 solutions, but feel free to check!